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RESEARCH ARTICLE

On Fuzzy Supra I-Open Sets and Fuzzy Supra I-Continuous Functions

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ABSTRACT : In this paper, we introduce and investigate a new class of sets and functions between fuzzy topological spaces called fuzzy supra I-open sets and fuzzy supra I-continuous functions respectively. Further Fuzzy Supra I-open sets are used for relations and certain properties investigated.

KEYWORDS: Fuzzy Supra I-Open set, Fuzzy Supra I-continuous functions, Fuzzy Supra topological spaces.

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1. INTRODUCTION

Fuzzy supra topological spaces and fuzzy supra-continuous functions introduced and studied in 1987 [1]. This class of sets contained in the class of β −open sets and contains all semi open sets and all pre-open sets [2]. The supra topological spaces and studied S-continuous maps and S^*-continuous maps were introduced in 1983 [2]. Now, in the present study, we introduce the concept of Fuzzy supra I-open set and fuzzy supra I-continuous functions and investigate several properties for these classes of functions. Further research for fuzzy supra I-open sets and fuzzy supra I-closed sets used fuzzy supra Ideal Baire Spaces will be discussed.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. The basic operations on fuzzy sets are taken as usual.

Definition 2.1[1]: A collection T^* of fuzzy open sets in a set X is called a fuzzy supra topology on X if the following conditions are satisfied:

$$(1) 0_X, 1_X \in T \quad and \quad (2) A_i \in T \Rightarrow \vee A_i \in T$$

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of a fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp.fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS(X) (resp. FSCS(X)).

Remark 2.2 [3]:

(1) Every FTS is a FSTS.
(2) If (X, T^*) is an associated FSTS with the FTS (X, T) (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS (X, T) is fuzzy supra open (closed) set in the FSTS (X, T^*).

Theorem 2.3 [3]: For any fuzzy sets A and B in a FSTS (X, T^*).

(1) $A \in FSCS(X) \Leftrightarrow Cl^*(A) = A, \ \ A \in FSOS(X) \Leftrightarrow Int^*(A) = A;$
(2) $A \leq B, \Rightarrow Cl^*(A) \leq Cl^*(B) and\ Int^*(A) \leq Int^*(B);$
(3) $Cl^*(Cl^*(A)) = Cl^*(A)\ and\ Int^*(Int^*(A)) = Int^*(A);$
(4) $Cl^*(A \vee B) \geq Cl^*(A) \vee Cl^*(B);$
(5) $Cl^*(A \wedge B) \leq Cl^*(A) \wedge Cl^*(B);$
(6) $Int^*(A \vee B) \geq Int^*(A) \vee Int^*(B);$
(7) $Int^*(A \wedge B) \leq Int^*(A) \wedge Int^*(B);$
(8) $Cl^*(A^C) = (Int^*(A))^C, Int^*(A^C) = (Cl^*(A))^C.$

Definition 2.4 [8]: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as

$$Cl^*(A) = \wedge \{B : A \leq B, B\ is\ a\ fuzzy\ supra\ closed\ set\ in\ X\},$$

$$Int^*(A) = \vee \{B : B \leq A, B\ is\ a\ fuzzy\ supra\ open\ set\ in\ X\}.$$

Remark 2.5 [8]:

(1) The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A.
(2) The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set containing in A.
(3) If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X, then

$$Int(A) \leq int^*(A) \leq A \leq Cl^*(A) \leq Cl(A).$$

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in the following theorem.

Definition 2.6 [8]: A fuzzy topology T on a set X is a family of fuzzy sets in X such that

(1) $0_X, 1_X \in T;$

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(2) $A, B \in T \Rightarrow A \wedge B \in T$ and

(3) $A_i \in T \Rightarrow \vee A_i \in T$

The pair (X, T) is called a fuzzy topological space (FTS). The elements of T are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.7 [12]: The closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$$Cl(A) = \wedge \{B : A \leq B, B \text{ is a fuzzy closed set in } X\},$$

$$Int(A) = \vee \{B : B \leq A, B \text{ is a fuzzy open set in } X\}.$$

I. FUZZY SUPRA I-OPEN SETS

In this section, we introduce a new class of generalized open sets called fuzzy supra I-open sets and study some of their properties.

Definition 3.1: Let (X, T^*) be a fuzzy supra topological space. A set A is called fuzzy supra I-open set if $A \leq fs\ int(fs\ cl(A))$. The complement of a fuzzy supra I-open set is called a fuzzy supra I-closed set.

Proposition 3.2: Every fuzzy supra open set is fuzzy supra I-open set.

Proof: Let A be a fuzzy supra open set in (X, T^*). Since, $A \leq fs\ cl(A)$, then $fs\ int\ (A) \leq fs\ int\left(fs\ cl(fs\ int(A))\right)$.

Hence $A \leq fs\ int\left(fs\ cl\left(fs\ int(A)\right)\right)$.

The converse of the above proposition need not be true as shown by the following example.

Example 3.3: Let (X, T^*) be a fuzzy supra topological space. Where $X = \{a, b, c\}$ and $T^* = \{X, \emptyset, \{a\}, \{a, b\}, \{b, c\}\}$. Here $\{a, c\}$ is a fuzzy supra I-open set, but it not a fuzzy supra open.

Proposition 3.4:

(i) Finite union of fuzzy supra I-open sets is always a fuzzy supra I-open.

(ii) Finite intersection of fuzzy supra I-open sets may fail to be a fuzzy supra I-open.

Proof: (i) Let A and B be two fuzzy supra I-open sets. The $A \leq fs\ int\left(fs\ cl\left(fs\ int(A)\right)\right)$ and $B \leq fs\ int\left(fs\ cl\left(fs\ int(B)\right)\right)$. Then $A \cup B \leq fs\ int\left(fs\ cl\left(fs\ int(A \cup B)\right)\right)$. Therefore $A \cup B$ is fuzzy supra I-open set.

(iii) Let (X, T^*) be a fuzzy supra topological space. Where $X = \{a, b, c, d\}$ and $T^* = \{X, \emptyset, \{a\}, \{a, b\}, \{b, c\}\}$. Here $\{a, b\}, \{b, c\}$ are fuzzy supra I-open sets but their intersection is not a fuzzy supra I-open set.

Proposition 3.5:

(i) Finite intersection of fuzzy supra I-closed sets is always fuzzy supra I-closed.

(ii) Finite union of fuzzy supra I-closed set may fail to be fuzzy supra I-closed set

Proof:

(i) This proposition follows from proposition 3.4.

(ii) Let (X, T^*) be a fuzzy supra topological space where $X = \{a, b, c, d\}$ and $T^* = \{X, \emptyset, \{a\}, \{a, b\}, \{b, c\}\}$. Here $\{c, d\}, \{a, d\}$ are fuzzy supra I-closed sets, but their union is not a fuzzy supra I-closed set.

Definition 3.6: The fuzzy supra I-closure of a set A is denote by fuzzy supra $Icl(A)$ and defined as, $fs\ I\ cl(A) = \cap \{B : B \text{ is a fuzzy supra } I - \text{closed set and } A \leq B\}$.

The fuzzy supra I-interior of a set is denoted by fuzzy supra $I\ int(A)$ and defined as, $fs\ I\ int(A) = \cup \{B : B \text{ is a fuzzy supra } I - \text{open set and } A \geq B\}$.

Remarks 3.7: It is clear that fuzzy supra $I\ int(A)$ is a fuzzy supra I-open set and fuzzy supra $Icl(A)$ is a fuzzy supra I-closed set.

Proposition 3.8:

(i) $X - fs\ I\ int(A) = fs\ I\ cl(X - A)$

(ii) $X - fs\ I\ cl(A) = fs\ I\ int(X - A)$

Proof: Obvious.

Proposition 3.9:

(i) $fs\ I\ int(A) \cup fs\ I\ int(B) = fs\ I\ int\ (A \cup B)$

(ii) $fs\ I\ cl(A) \cap fs\ I\ cl(B) = fs\ I\ int\ (A \cap B)$

Proof: Obvious.

I. FUZZY SUPRA I-CONTINUOUS FUNCTIONS

In this section, we introduce a new type of continuous functions called a fuzzy supra I-continuous function and obtain some of their properties and characterizations.

Definition 4.1: Let (X, τ) and (Y, σ) be two fuzzy topological space and T^* be associated fuzzy supra topology with τ. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a fuzzy supra I-continuous function if the inverse image of each open set in y is a fuzzy supra I-open set in x.

Proposition 4.2: Every fuzzy continuous function is fuzzy supra I-continuous function.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy continuous function. Therefore $f^{-1}(A)$ is an fuzzy open set in X for each fuzzy open set A in Y. But, T^* is associated with τ. That is, $\tau \subset T^*$. This implies $f^{-1}(A)$ is an fuzzy supra open in X. Since fuzzy supra open is fuzzy supra I-open this implies that $f^{-1}(A)$ is fuzzy supra I-open in X. Hence f is a fuzzy supra I-continuous function.

The converse of the above proposition is not true as shown in the following example.

Example 4.3: Let $X = \{a, b, c\}$ and $T = \{X, \emptyset, \{a, b\}\}$ be a fuzzy topology on X. The fuzzy supra topology T^* is defined as follows, $T^* = \{X, \emptyset, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (X, \tau)$ be a function defined as follows $f(a) = a, f(b) = c, f(c) = b$. The inverse image of the fuzzy open set $\{a, b\}$ is $\{a, c\}$ which is not an fuzzy open set but if a fuzzy supra I-open set.. Then f is fuzzy supra I-continuous but it is not fuzzy continuous.

Proposition 4.4: Let (X, τ) and (Y, σ) be two fuzzy topological spaces. Let f be a function from X into Y. Let T^* be associated fuzzy supra topology withτ. Then the following equivalent.

(a) f is fuzzy supra I-continuous.

(b) The inverse image of fuzzy closed set in Y is a fuzzy supra I-closed set in X.

(c) $fs\ I\ cl\left(f^{-1}(A)\right) \leq f^{-1}\left(cl(A)\right)$ for every fuzzy set A in Y.

(d) $f(fs\,I\,cl(A)) \le cl\,(f(A))$ for every fuzzy set A in X.

(e) $f^{-1}(int(B)) \le fs\,I\,int\,(f^{-1}(B))$ for every fuzzy set B in Y.

Proof: **(a)** $\Rightarrow$ $(\boldsymbol{b})$**:** Let A be a fuzzy closed set in Y, then $Y - A$ is fuzzy open in Y. Thus, $f^{-1}(X - A) = X - f^{-1}(A)$ in fuzzy supra I-open in X. It follows that $f^{-1}(A)$ is a fuzzy supra I-closed set of X.

(b) $\Rightarrow$ $(\boldsymbol{c})$**:** Let A be a fuzzy set in X. Since $cl(A)$ is fuzzy closed in Y, then it follows that $f^{-1}(cl(A))$ is fuzzy supra I-closed set in X. Therefore, $f^{-1}(cl(A)) = fs\,I\,cl\,f^{-1}(cl(A)) \ge fs\,I\,cl\,(f^{-1}(A))$.

(c) $\Rightarrow$ $(\boldsymbol{d})$**:** Let A be a fuzzy set in X. By (c) we obtain, $f^{-1}\left(cl(f(A))\right) \ge fs\,I\,cl(A)$ and hence $f(fs\,I\,cl(A)) \le cl(f(A))$.

(d) $\Rightarrow$ $(\boldsymbol{e})$**:** Let $f(fs\,I\,cl(A)) \le cl\,f(A)$ for every fuzzy set A in X. Then $fsIcl(A) \le f^{-1}\left(cl(f(A))\right), X - fsIcl(A) \ge X - f^{-1}\left(cl(f(A))\right)$ and $fs\,I\,int\,(X - A) \ge f^{-1}\left(int(y - f(A))\right)$. Then $fs\,I\,int(f^{-1}(B)) \ge f^{-1}(int(B))$. Therefore, $f^{-1}(int(B)) \le fs\,I\,int(f^{-1}(B))$, for every B *in* Y.

(e) $\Rightarrow$ $(\boldsymbol{a})$**:** Let A be a fuzzy open set in Y. Therefore $f^{-1}(int(A)) \le fs\,I\,int(f^{-1}(A))$, hence $f^{-1}(A) \le fs\,I\,int\,(f^{-1}(A))$. But by other hand we know that, $fsI\,int\,(f^{-1}(A)) \le f^{-1}(A)$. Then $f^{-1}(A) = fs\,I\,int\,(f^{-1}(A))$. Therefore , $f^{-1}(A)$ is a fuzzy supra I-open set.

Proposition 4.5: Let (X,τ) and (Y,σ) be two fuzzy topological spaces. Let τ^* *and* σ^* be associated fuzzy supra topologies with τ and σ respectively. Then $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy supra I –continuous function, if one of the following holds.

(1) $f^{-1}(fs\,I\,int(B)) \le int\,(f^{-1}(B))$ for every fuzzy set B in Y.

(2) $cl\,(f^{-1}(B)) \le f^{-1}(fs\,Icl(B))$ for every fuzzy set B in Y.

(3) $fcl(A) \le fs\,I\,cl(f(A))$ for any fuzzy set A in X.

Proof: Let B be any fuzzy open set of Y, if condition (1) is satisfied, then $f^{-1}(fs\,I\,int(B)) \le int\,(f^{-1}(B))$. we get $f^{-1}(B) \le int\,(f^{-1}(B))$. Therefore $f^{-1}(B)$ is fuzzy supra open set. Every fuzzy supra open set is fuzzy supra I-open set. Hence f is a fuzzy supra I-continuous function.

If condition (2) is satisfied, then we can easily prove that f is a fuzzy supra I-continuous function.

If condition (3) is satisfied and B is any fuzzy open set of Y, then $f^{-1}(B)$ is a fuzzy set in X and $f\left(cl\left(f^{-1}(B)\right)\right) \le fsI\,cl\left(f\left(f^{-1}(B)\right)\right)$. This implies $f\left(cl\left(f^{-1}(B)\right)\right) \le fsI\,cl(A)$. Hence f is a fuzzy supra I-continuous function.

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